Model-Free Control for Real-Time Dynamic Traffic Routing Problems

Hassane Abouaïssa

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***Abstract*— The aim of real-time Dynamic Trafic Routing (DTR) is to find the time-dependent split variables at the diversion point for routing the incoming traffic flow onto the alternate routes in order to reach a user equilibrium traffic pattern. This paper exploits the recent advances on model-free control and proposes a robust and easy implementable algorithm to deal with the DTR problem in the case of non-recurrent congestions. The provided simulations show the relevance of the developed approach.**

I. INTRODUCTION

Although the extension of the existing road networks, adding lanes and designing alternative new infrastructures are considered as a natural solution against the ever growing traffic jam problems, they cannot be always utilized due to the lack of space and their expensive costs. Therefore, transportation engineers and researchers have focused on the development of dynamics traffic managements approaches, resulting of a huge literature devoted to these problems (See, *e.g.*, [22], [23], [28], [29], [33], [34], [36], [43], and references therein).

Indeed, at the network level, dynamic traffic flow management represents a valuable means to improve the highway throughput and to ensure a efficient, safe and less polluting transportation of goods and persons [42]. It also contributes to a large reduction of direct and indirect costs.

Several actions have been developed in the frame of intelligent transportation systems. Mandatory actions like ramp metering and dynamic speed limit, are the most implemented control measurements. Route guidance and dynamic traffic routing, although that are a non mandatory actions, aim to provide a reliable information to the users in term of travel time, and are more adapted in the case of non recurrent congestions.

Dynamic Traffic Routing (**DTR**) and (**DTA**, Dynamic traffic assignment) aim to achieve a user-equilibrium traffic pattern via Dynamic Variable Message Sign (DVMS) [21] via the calculation of time-dependent split defined as a control variable.

With the advent of Intelligent Transportation Systems (**ITS**), several algorithms have been developed in order to provide an efficient framework for real time control for such problem. Among these, [18] and [30] have proposed a strategies based on the expert systems. [39] have developed a solution for such problem using optimization techniques. A linear quadratic, and nonlinear optimization techniques for Dynamic Traffic Assignment (**DTA**) have been developed by [37] and [35]. Most of these approaches are designed for off-line control and do not take into account real-time traffic flow variation. Using the mass conservation law and the fundamental diagram proposed by Greenshields [17], Kachroo and Özbay have designed several on-line diversion algorithms starting from feedback, sliding mode control to fuzzy control laws, [21] [24],[25], (See also [26]). Although these algorithms seem to be efficient, the use of Greenshields fundamental prevents taking into account the whole traffic flow phenomena. Previous work, [4] has proposed an algorithm based on the concept of differential flatness, successfully applied to DTR problem.

Nevertheless, most of the proposed algorithms rest on the use of macroscopic traffic model. Although the technical literature on macroscopic traffic flow modeling is vast and increasing in an accelerated pace, rigorous model validation exercises using real traffic data are surprisingly sparse. Given the largely empirical character of the proposed models, the lack of validation efforts is a shortcoming that cannot be sufficiently emphasized [38]. Moreover, application of such models to DTR/DTA leads to very complex expressions at network level.

This paper develops the following point which is quite new in traffic engineering and therefore in DTR problem also (See e.g., [1], [2], [3], for the first applications to ramp-metering, and [20], for successful implementation). It concerns the concept of *Model-free* control, or *MFC*, and the corresponding *intelligent PID controllers*, or *iPID*, which were introduced by Fliess and Join (see [8], [9], [11]) and yield a straightforward control strategy where:

- the need of any global mathematical description of the traffic flow becomes unnecessary,
- robustness with respect to quite strong disturbances is provided,
- the implementation and the tuning of the feedback loop become obvious.

The paper is structured as follow. Section II, recalls the principle of the dynamic traffic routing problem and the used, for simulation purposes, macroscopic model. Section III presents an overview of model-free control and its main elements which are employed here, for DTR problem. The main idea rests on the following

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principle. The unknown global equations, which cannot be derived for complex systems from simple physical laws, are replaced by a very simple differential equation, in general of first order, which is only valid during a very short time window. The real-time adaptation of this last system is achieved via a parameter identification technique, which goes back to ([12], [13]). It yields a time-varying term which contains the unknown parts of the system as well as the unknown perturbations, without making any difference between them. It yields a proportional-integral controller which is "intelligent", *i.e.*, it is endowed with an additive term which

- is deduced from the previous estimation,
- alleviates the unknown parts and perturbations.

We are left to a linear first order system. The tuning of the iIP is straightforward. Section IV, deals with the application of MFC to DTR problem. Computer simulations with real data are developed in Section V. A few concluding remarks are stressed in Section VI.

II. Dynamic traffic routing problem

A. Review of traffic models

Since the early works of Lighthill, Whitham and Richards (LWR), [32], [41], several models have been proposed in order to deal with the thorny problem of traffic flow behavior. Their classification, which is of utmost importance allows a better understanding of the complex phenomena of traffic in order to implement control actions to eliminate or at least minimize the effects of congestion. Such classification can be performed according to the degree of granularity which leads to microscopic, mesoscopic, and macroscopic set of models. Most of the conducted works in the field of intelligent transportation systems underlines that macroscopic models are more adapted and useful one for the traffic flow monitoring, planification and dynamic management [36], [19].

Macroscopic models are based on the conservation law and are derived by analogy with fluid dynamic and consider the following partial differential equation relating the traffic density $\rho(x,t)$ (in a number of vehicles per kilometers (veh/km)) and the traffic flow $q(x,t)$ expressed in term of a number of vehicles per hour (veh/h).

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0 \tag{1}$$

The space discretization of (1) assumes that the highway is subdivided into a set of segments. Therefore, the conservation equation reads:

$$\dot{\rho}_{i,j}(t) = \frac{1}{L_i}\left[q_{i,j-1}(t) - q_{i,j}\right] \tag{2}$$

where

- $(i,j) = ((1,1),(1,2),\ldots,(2,n_1),(2,1),(2,2),\ldots,(2,n_2))$.
- L_i is the section length and $\rho_{i,j}$ represents the traffic density at section j of the route i.

B. DTR formulation

The main objective of dynamic traffic routing is to reach a user-equilibrium. Let us stress that the control action is mainly informative and it is not a mandatory. The success of the control depends on the driver's compliance. The implementation of such algorithm may follows the steps below:

- For each road, calculate the travel time following [5]. This calculation is, usually achieved via tracking vehicles.
- Based on the desired trajectory (here equal travel time), the control algorithm calculates the split rate β which defines the road allowing an optimal cost. The cost in traffic engineering is already related to the comfort, safety, or travel time of the possible alternative routes to the desired destination.
- At each step the DVMS shows the travel time realized by the users (See [27], for more explanation).

In order to formulate the DTR problem, consider the simple highway depicted in Figure 1.

At each of the two alternate routes of the example, the relationship between the traffic density ρ_i and traffic flow q_i includes the mean speed v_i as follow:

$$q_i(t) = \rho_i(t) v_i(t) \lambda_i \tag{3}$$

where v_i is defined as a nonlinear expression (See [34]):

$$v_i = v_{fi} \exp\left[\left(-\frac{1}{a}\right)\left(\frac{\rho_i}{\rho_{ci}}\right)^a\right] \tag{4}$$

ρ_{ci} represents the critical density and a, a model parameter. v_{fi}, the free-flow speed.

The aim of the DTR problem is to synthesis a control law which ensure to reach a user equilibrium traffic pattern. Consider $\beta(t) \in [0,\ 1]$, the control variable that ensures this user equilibrium. This will be made by minimizing the total travel time TT.

$$\begin{cases} \beta(t) q_e(t) = & q_{1,0}(t) \\ (1-\beta) q_e(t) = & q_{2,0}(t) \end{cases} \tag{5}$$

$q_e(t)$ is the incoming measured flow.
As in Kachroo and Özbay [21], the dynamic routing problem can be expressed as a problem of minimization of the objective function in (6) by finding the split rate β_0;

$$J(\beta) = \int_0^{t_f} \left[\sum_{i=1}^{m} TT(\rho_i) - \sum_{m+1}^{m+p} TT(\rho_i)\right]^2 dt \tag{6}$$

where, TT is the travel time function and t_f the final time.

The same raisonnement may be applied for a highway with n alternate routes. In this case, the whole network subdivided into several segments is described by (2), where, $(i,j) = ((1,1),\ldots,(1,n_1),(2,1),(2,2),\ldots,(2,n_2),(n,1),\ldots,(n,n_n))$.

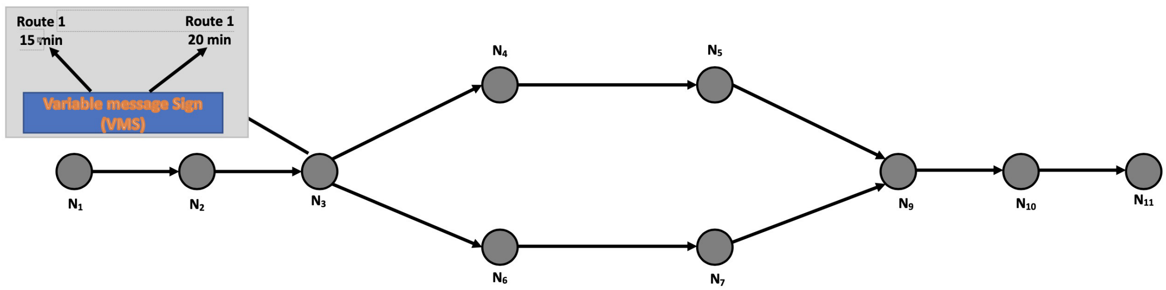

Fig. 1. Simple example of two alternate routes

The problem consists then on finding a set of split variables, β_{n-1}, where, $\sum_{i=1}^{n-1} \beta^i = 1$ which minimize the total travel time TT, (See [21], for the optimal formulation of the DTR problem for a general case)

$$\begin{cases} \beta(t)q_e(t) = & q_{1,0}(t) \\ \beta(t)q_e(t) = & q_{2,0}(t) \\ \vdots & \\ (1-\beta_1-\beta_2-,\ldots,-\beta_{n-1})q_e(t) = & q_{n,0}(t) \end{cases} \tag{7}$$

Notice that the travel time function TT_i for each alternate route i is defined by the following expression:

$$TT_i = \frac{L_i}{v_i} \tag{8}$$

The control variable can be obtained thanks to the first derivative of the equation (8). It is not difficult to see that for a large network, the obtained equations providing different split rates β_i become very complexes. Using the recently introduced model-free control theory allows, among other, to overcome these heavy stages of the control design. The following section provides the basic principle of this concept.

III. Model-free control: an overview

Most of the developed control strategies are model-based. Nevertheless, obtaining a reliable mathematical model tacking into account all physical and dynamical behaviors is always a difficult task. To overcome these problem, model-free control has been recently introduced where the physical representation is replaced by a purely numerical model called "phenomenological model" [14], (See e.g. [16], [9] for a thorough presentation) Such ultra-local model involves very few parameters which are estimated, online via algebraic methods of identification. The control action is then derived and tuned easily by this numerical model.

A. Model-free control principle

For simplicity's sake, consider a single-input single-output (SISO) system $\mathfrak{S}$ which is unknown. Replace the unknown "global" model by the following "phenomenological" one:

$$\boxed{y^{(\nu)} = F + \alpha u} \tag{9}$$

which is

- $\alpha \in \mathbb{R}$ is a non-physical constant parameter that has no a priori precise numerical value. This parameter is chosen by the engineer such as αu and $y^{(\nu)}$ are of equivalent magnitude
- The model (9) is continuously updated.
- F which subsumes all the unknown parts of the system including perturbations is estimated as in subsection III.B.

Remark 3.1: Notice that (9) should not be confused with a "black-box" identified model.

B. Estimation of F

Under a weak integrability condition, F in (9) may be "well" approximated by a piecewise constant function $\hat{F}$. Let us summarize two types of the techniques borrowed from [13].

1) Rewrite (9) with $\nu = 1$ in operational domain (See [44])

$$sY(s) - y(0) = \frac{\Phi}{s} + \alpha U(s) \tag{10}$$

where Φ is a constant. To get rid of the initial condition $y(0)$, multiply both sides on the left by $\frac{d}{ds}$.

$$\frac{\Phi}{s^2} = -Y(s) - s\frac{dY(s)}{ds} + \alpha\frac{U(s)}{ds} \tag{11}$$

For noise attenuation, multiply both sides on the left by s^{-2}, which is equivalent, in time domain to a lowpass filter. It yields in time domain the real time estimate, thanks to the equivalence between $\frac{d^k}{ds^k}$, $k \geq 1$ and the multiplication by $(-t)^k$

$$\hat{F} = \frac{6}{\tau^3}\int_{t-\sigma}^{t} [(\tau - 2\sigma)y(\sigma) + \alpha\sigma(\tau-\sigma)u(\sigma)]\,d\sigma \tag{12}$$

where $\sigma > 0$ might be quite small, [15]. Notice that in practice, this integral may be replaced by classic digital filter.

2) For the second technique, close the loop with the intelligent proportional controller *iP*. It yields:

$$\hat{F} = \frac{1}{\tau}\left[\int_{t-\tau}^{t} (\dot{y}^\star - \alpha u - K_p e)\,d\sigma\right] \tag{13}$$

C. Intelligent controller

The knowledge of F permits the calculation of the control variable $u(k)$ at the sampling period k using equation (14). This calculation is a simple cancellation of the nonlinear term F in addition to a closed loop tracking of a desired trajectory $t \to y^\star$ [16]:

$$\boxed{u(k) = -\underbrace{\frac{[F(k)]_e}{\alpha}}_{\text{NL Cancellation}} + \underbrace{\frac{\dot{y}^{(\nu)\star}(k)}{\alpha} + K_p e + K_i \int e + K_d e}_{\text{Closed loop tracking}}} \tag{14}$$

where

- $y^\star$ is the output reference trajectory, which is obtained via the differential flatness principle (see, e.g., [6], [40]),
- $e = y - y^*$ is the tracking error,
- K_p, K_i and K_d are parameters to be tuned.

Remark 3.2: The term $\frac{[F(k)]_e}{\alpha} + \frac{\dot{y}^{(\nu)\star}}{\alpha}$ represents the "nominal control" in the "Flatness-based control".

Remark 3.3: It is important to emphasize that, in contrast with the classical PID controllers, here, no identification procedure is needed since the whole structural information is containing in the term F which is canceled [10].

IV. Application to DTR problem

As mentioned above, the travel time TT is defined with the equation (8) and the DTR problem is solved using the first derivative of this entity. In terme of model free control, such equation is replaced by the following "phenomenological model"

$$\dot{TT}(t) = F(t) + \alpha\beta_i(t) \tag{15}$$

where β_i represents the split rate at each controlled diversion point. Following the MFC principle, this control variable reads

$$\beta_i = \frac{1}{\alpha}\left(-[F]_e + \dot{TT}^\star - K_P e\right) \tag{16}$$

where

- $TT^\star$ is the reference trajectory.
- $e = TT - TT^\star$ represents the tracking error.
- K_P is a positive gain.

Remark 4.1: It should be notice that, equation (15) is here first order, a simple iP regulator is enough to ensure convergence of the error to zero [16]. Indeed, contrarily to classic feedback control, since the integral effect is included in the term $[F]_e$,[1] there is no need of an integral controller in order to ensure stabilization and convergence of the error to zero.
The estimation of F is provided thanks to (12) , (13) or using the following expression:

$$\boxed{[F]_e = [\dot{TT}(k)]_e - \alpha\beta_i(k-1)} \tag{17}$$

where

- k is the sampled time
- $[\bullet]_e$ indicates an estimate of $\bullet$.

V. Numerical simulations

For simulation purpose, consider the simple network depicted in Fig. 1. For this example, assume that the two alternate routes are decomposed into three segments with two lanes. In addition it is assumed a full drivers compliance. Sections at the origin between the nodes N_1 and N_3 and at the destination between N_9 and N_{11} are with four lanes.

The conducted simulations rest of the second order macroscopic model called "METANET" (See e.g. [31] for more explanation about this model). In addition to (2), (3) and (4), the model takes into account the speed dynamic:

$$\begin{aligned} v_i(t) = \; & \tfrac{1}{\tau}\left[V_e(\rho_i) - v_i(t)\right] + \tfrac{1}{L_i} v_i(t)\left[v_{i-1}(t) - v_i(t)\right] \\ & - \tfrac{\eta}{\tau L_i}\tfrac{\rho_{i+1}(t) - \rho_i(t)}{\rho_i(t) + \upsilon} \end{aligned} \tag{18}$$

where, $V_e(\rho_i)$ is the May fundamental diagram defined by (4). where, τ, η and υ are the model parameters. The traffic demand represented in figure 2, is provided via a loop detector every 20s between 5 am and 8 pm, in order to take into account the peak hours that include congestions. Notice that the provided real data are often corrupted by noises.

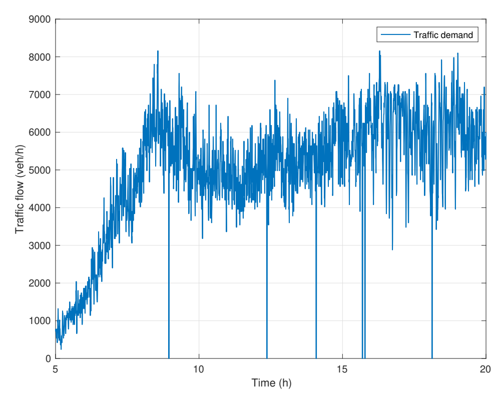

Fig. 2. Traffic demand at the origin

In the no control case, Fig.3 and Fig.4 display the densities and the flow evolutions in the two alternate routes. The difference of travel time is shown in Fig. 5.

Fig. 6 shows the travel time of each of the two alternate routes in the control case and confirms the performance of the proposed controller that is able to ensure a perfect routing at the diversion point as depicted also in Fig. 7.

Notice that with equal travel time in both route 1 and route 2 the controller allows to minimize the difference of travel time which allows an equal traffic flows and densities in both routes (See. Fig. 8 and Fig. 9).

[1] This fact comes from the estimation of F via the algebraic method which leads to a set of iterated integrals.

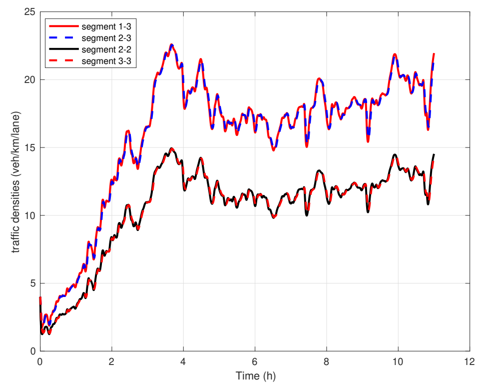

Fig. 3. Traffic densities in the two alternate routes

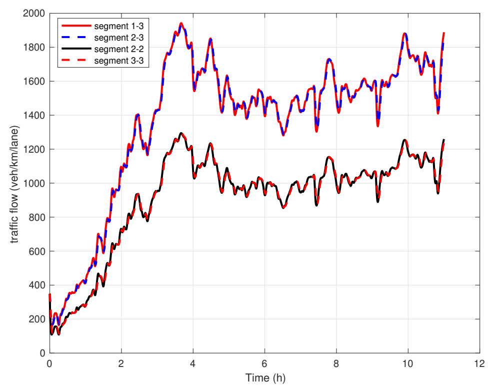

Fig. 4. Traffic flow in the two alternate routes

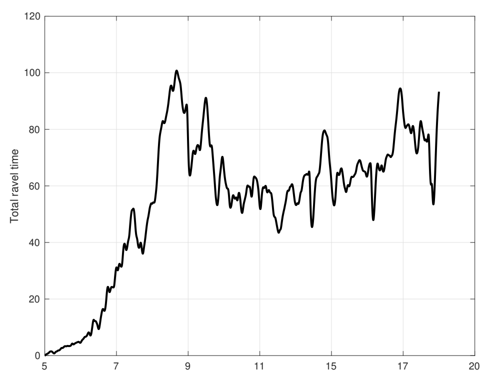

Fig. 5. Difference of travel time

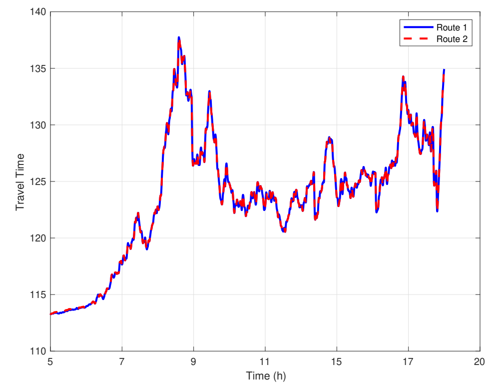

Fig. 6. Travel time of each alternate route

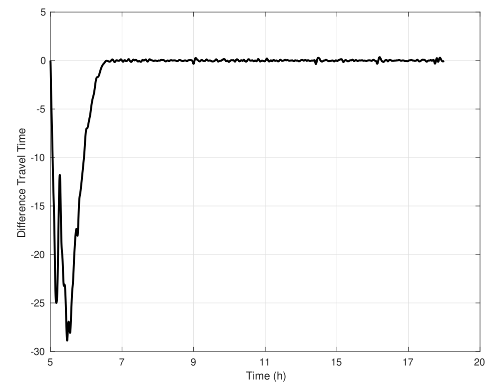

Fig. 7. Difference of travel times

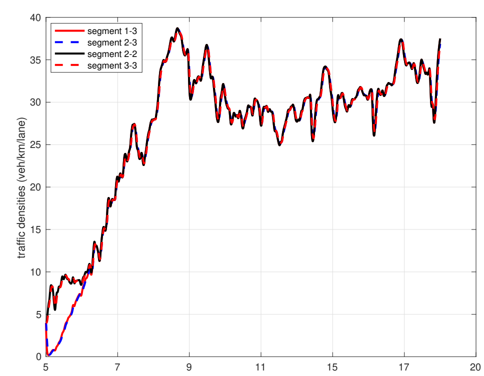

Fig. 8. Traffic densities in the two alternate routes

VI. CONCLUSIONS

Based on a simple but realistic example, this paper shows the efficiency of model-free control to deal with the dynamic traffic routing problem. The main objective is to ensure a user-equilibrium pattern in order to solve the problem of non-recurrent congestions. The proposed approach which is based on algebraic methods of identification provides a solid alternative to deal with dynamic guidance and traffic routing.

Furthers works will provide an integrated approach using real-time ramp metering and route guidance in order to efficiently minimize the recurrent and non-recurrent traffic congestion at networks level.

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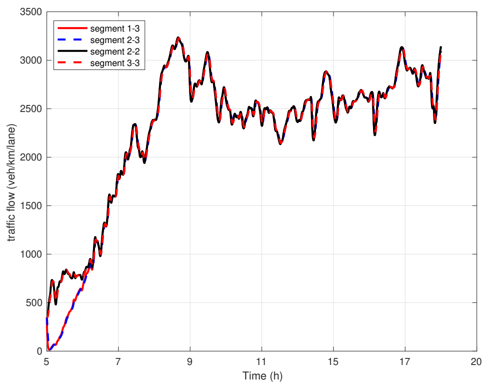

Fig. 9. Traffic flow in the two alternate routes

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